Describing Data: Numerical Measures

Chapter 3

McGraw-Hill/Irwin

Learning Objectives

LO1 Explain the concept of central tendency.
LO2 Identify and compute the arithmetic mean.
LO3 Compute and interpret the weighted mean.
LO4 Determine the median.
LO5 Identify the mode.
LO6 Calculate the geometric mean.
LO7 Explain and apply measures of dispersion.
LO8 Compute and interpret the standard deviation.
LO9 Explain the Empirical Rule.
L10 Compute the mean and standard deviation of grouped data.

LO1 Explain the concept of central tendency

Central Tendency - Measures of Location

- The purpose of a measure of location is to pinpoint the center of a distribution of data.
- There are many measures of location. We will consider five:
  1. The arithmetic mean,
  2. The weighted mean,
  3. The median,
  4. The mode

LO2 Identify and compute the arithmetic mean.

Characteristics of the Mean

- The arithmetic mean is the most widely used measure of location.
- Requires the interval scale.
- Major characteristics:
  - All values are used.
  - It is unique.
  - The sum of the deviations from the mean is 0.
  - It is calculated by summing the values and dividing by the number of values.

LO2

Population Mean

For ungrouped data, the population mean is the sum of all the population values divided by the total number of population values:

POPULATION MEAN $$\mu = \frac{\Sigma X}{N}$$ **[3–1]**

where:

- μ represents the population mean. It is the Greek lowercase letter “mu.”
- N is the number of values in the population.
- X represents any particular value.
- Σ is the Greek capital letter “sigma” and indicates the operation of adding.
- ΣX is the sum of the X values in the population.

LO2

EXAMPLE – Population Mean

There are 42 exits on I-75 through the state of Kentucky. Listed below are the distances between exits (in miles).

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

Why is this information a population?
What is the mean number of miles between exits?

LO2

EXAMPLE – Population Mean

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11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

Why is this information a population?

This is a population because we are considering all the exits in Kentucky.

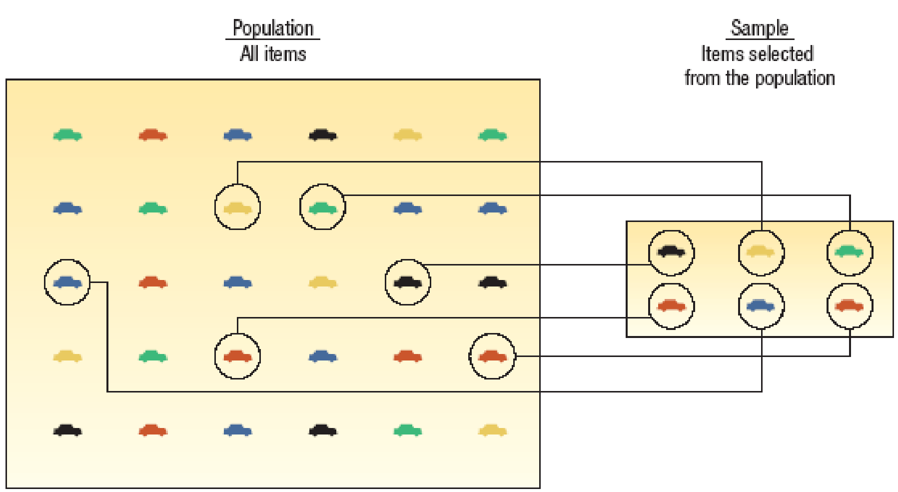
What is the mean number of miles between exits?

$$\mu = \frac{\Sigma X}{N} = \frac{11 + 4 + 10 + \cdots + 1}{42} = \frac{192}{42} = 4.57$$

Parameter Versus Statistics

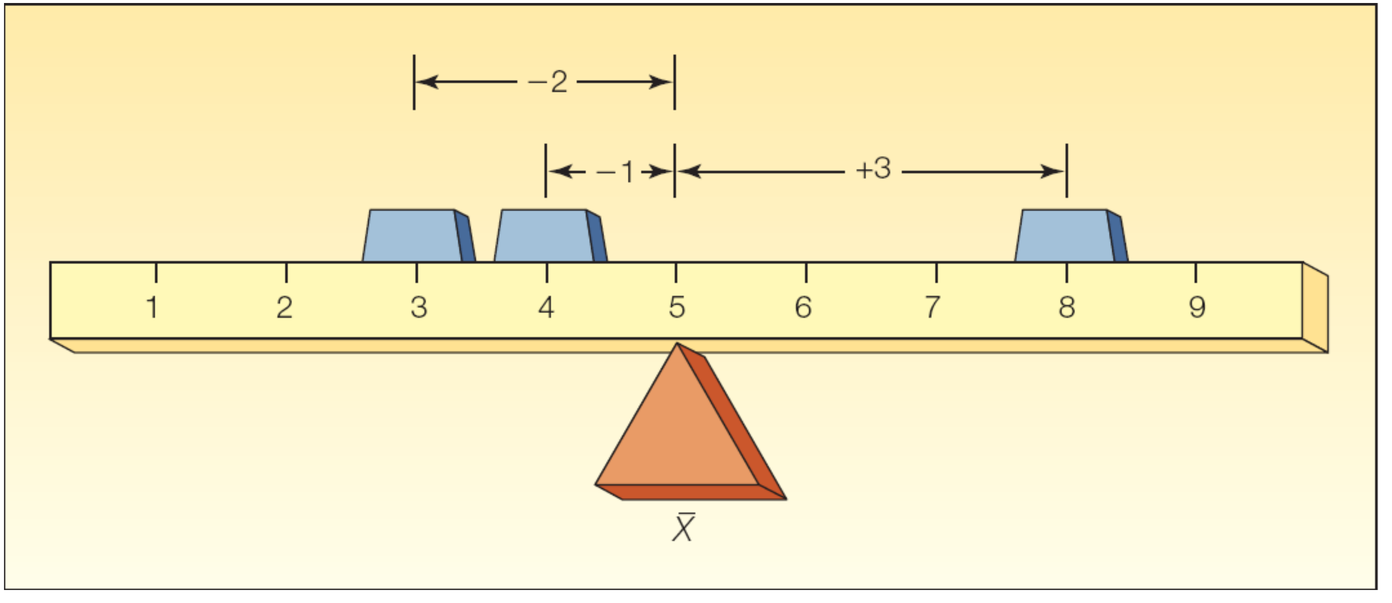
PARAMETER A measurable characteristic of a *population*.

STATISTIC A measurable characteristic of a *sample*.



Properties of the Arithmetic Mean

1. Every set of interval-level and ratio-level data has a mean.
2. All the values are included in computing the mean.
3. The mean is unique.
4. The sum of the deviations of each value from the mean is zero.



Sample Mean

- For ungrouped data, the sample mean is the sum of all the sample values divided by the number of sample values:

SAMPLE MEAN

$$\bar{X} = \frac{\Sigma X}{n} \quad [3\text{–}2]$$

where:

$\bar{X}$ is the sample mean. It is read "X bar."

n is the number of values in the sample.

EXAMPLE – Sample Mean

SunCom is studying the number of minutes used monthly by clients in a particular cell phone rate plan. A random sample of 12 clients showed the following number of minutes used last month.

90	77	94	89	119	112
91	110	92	100	113	83

What is the arithmetic mean number of minutes used?

$$\text{Sample mean} = \frac{\text{Sum of all values in the sample}}{\text{Number of values in the sample}}$$

$$\overline{X} = \frac{\Sigma X}{n} = \frac{90 + 77 + \cdots + 83}{12} = \frac{1170}{12} = 97.5$$

LO3 Compute and interpret the weighted mean

Weighted Mean

- The weighted mean of a set of numbers X_1, X_2, ..., X_n, with corresponding weights w_1, w_2, ...,w_n, is computed from the following formula:

WEIGHTED MEAN

$$\bar{X}_w = \frac{w_1X_1 + w_2X_2 + w_3X_3 + \cdots + w_nX_n}{w_1 + w_2 + w_3 + \cdots + w_n} \quad [3\text{–}3]$$

LO3

EXAMPLE – Weighted Mean

The Carter Construction Company pays its hourly employees $16.50, $19.00, or $25.00 per hour. There are 26 hourly employees, 14 of which are paid at the $16.50 rate, 10 at the $19.00 rate, and 2 at the $25.00 rate.

What is the mean hourly rate paid the 26 employees?

LO3

EXAMPLE – Weighted Mean

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What is the mean hourly rate paid the 26 employees?

$$\bar{X}_w = \frac{14(\$16.50) + 10(\$19.00) + 2(\$25.00)}{14 + 10 + 2} = \frac{\$471.00}{26} = \$18.1154$$

The Median

MEDIAN The midpoint of the values after they have been ordered from the smallest to the largest, or the largest to the smallest.

PROPERTIES OF THE MEDIAN

1. There is a unique median for each data set.
2. It is not affected by extremely large or small values and is therefore a valuable measure of central tendency when such values occur.
3. It can be computed for ratio-level, interval-level, and ordinal-level data.
4. It can be computed for an open-ended frequency distribution if the median does not lie in an open-ended class.

LO4

EXAMPLES - Median

The ages for a sample of five college students are:

21, 25, 19, 20, 22

Arranging the data in ascending order gives:

19, 20, 21, 22, 25.

Thus the median is 21.

The heights of four basketball players, in inches, are:

76, 73, 80, 75

Arranging the data in ascending order gives:

73, 75, 76, 80.

Thus the median is 75.5

The Mode

MODE The value of the observation that appears most frequently.

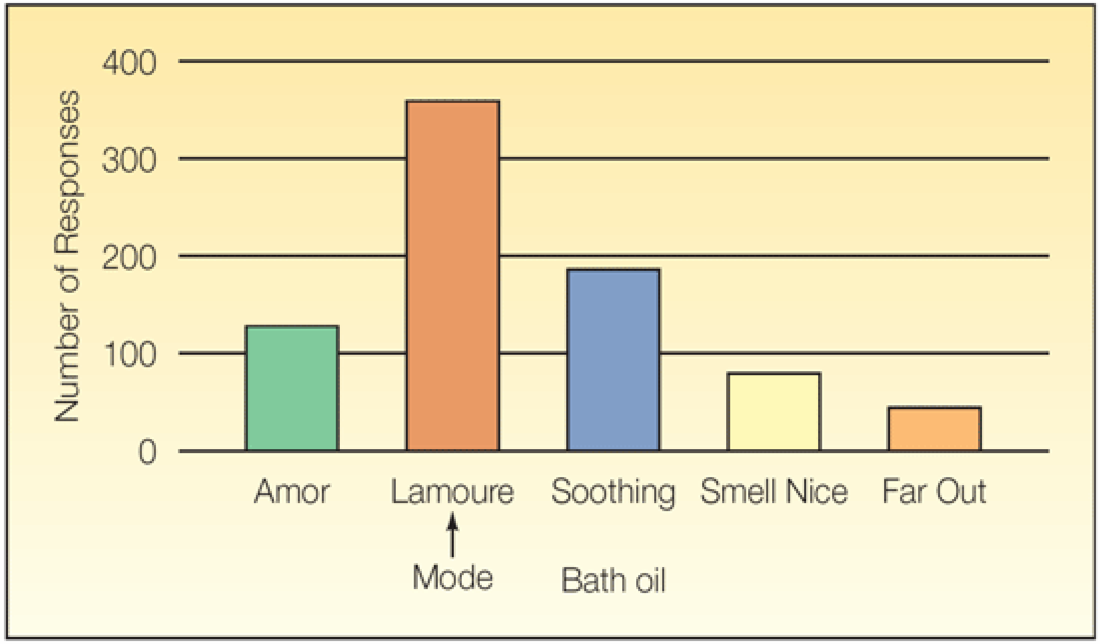


CHART 3–1 Number of Respondents Favoring Various Bath Oils

Example - Mode

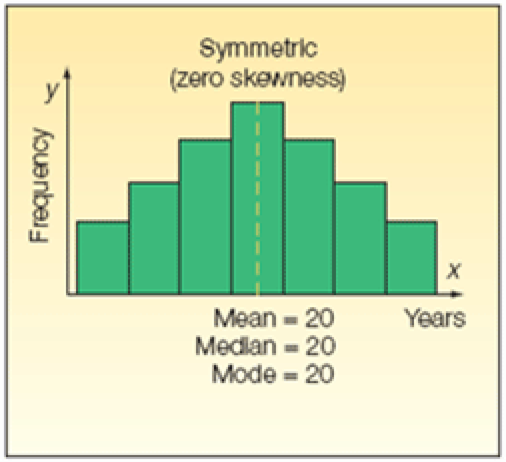
Using the data regarding the distance in miles between exits on I-75 through Kentucky. The information is repeated below. What is the modal distance?

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

Organize the distances into a frequency table.

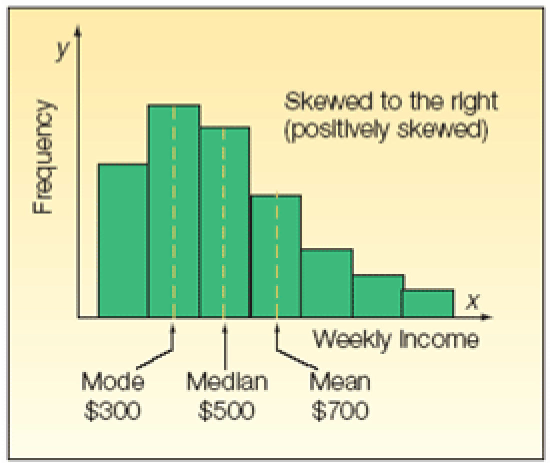
Distance in Miles between Exits	Frequency
1	8
2	7
3	7
4	3
5	4
6	1
7	3
8	2
9	1
10	4
11	1
14	1
Total	42

The Relative Positions of the Mean, Median and the Mode



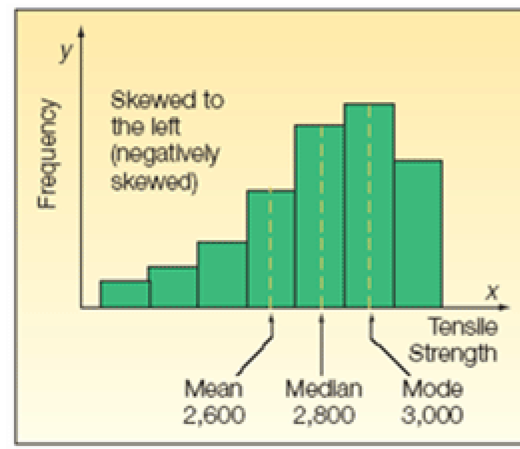
zero skewness

mode = median = mean



positive skewness

mode < median < mean



negative skewness

mode > median > mean

Dispersion

A measure of location, such as the mean or the median, only describes the center of the data. It is valuable from that standpoint, but it does not tell us anything about the *spread* of the data.

For example, if your nature guide told you that the river ahead averaged 3 feet in depth, would you want to wade across on foot without additional information? Probably not. You would want to know something about the variation in the depth.

A second reason for studying the dispersion in a set of data is to compare the spread in two or more distributions.

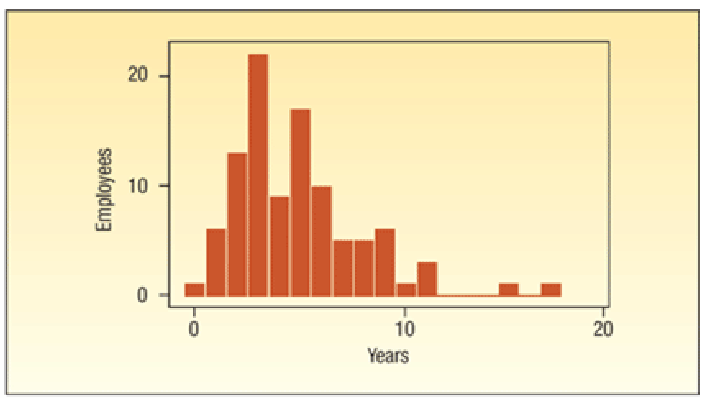


CHART 3–5 Histogram of Years of Employment at Hammond Iron Works, Inc.

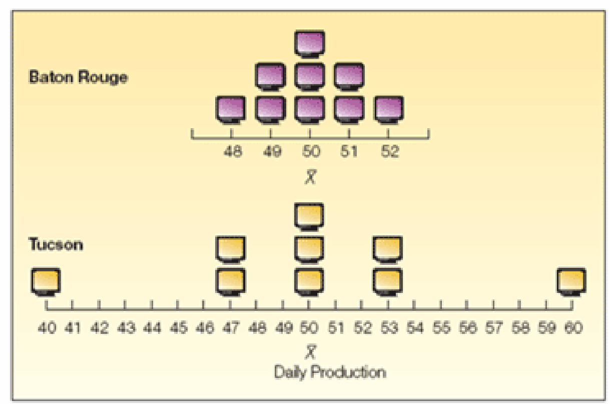


CHART 3–6 Hourly Production of Computer Monitors at the Baton Rouge and Tucson Plants

LO7

Measures of Dispersion

- Range

RANGE $\text{Range} = \text{Largest value} - \text{Smallest value}$ [3–6]

- Mean Deviation

MEAN DEVIATION $$MD = \frac{\Sigma|X - \overline{X}|}{n}$$ [3–7]

- Variance and Standard Deviation

POPULATION VARIANCE $$\sigma^2 = \frac{\Sigma(X - \mu)^2}{N}$$ [3–8]

POPULATION STANDARD DEVIATION $$\sigma = \sqrt{\frac{\Sigma(X - \mu)^2}{N}}$$ [3–9]

EXAMPLE – Range

The number of cappuccinos sold at the Starbucks location in the Orange Country Airport between 4 and 7 p.m. for a sample of 5 days last year were 20, 40, 50, 60, and 80. Determine the range for the number of cappuccinos sold.

$$\text{Range} = \text{Largest} - \text{Smallest value}$$
$$= 80 - 20 = 60$$

LO7

Mean Deviation

MEAN DEVIATION The arithmetic mean of the absolute values of the deviations from the arithmetic mean.

- A shortcoming of the range is that it is based on only two values, the highest and the lowest; it does not take into consideration all of the values.
- The **mean deviation** does. It measures the mean amount by which the values in a population, or sample, vary from their mean

MEAN DEVIATION

$$MD = \frac{\Sigma|X - \overline{X}|}{n} \quad [3\text{–}7]$$

where:

X is the value of each observation.
$\overline{X}$ is the arithmetic mean of the values.
n is the number of observations in the sample.
$||$ indicates the absolute value.

EXAMPLE – Mean Deviation

The number of cappuccinos sold at the Starbucks location in the Orange Country Airport between 4 and 7 p.m. for a sample of 5 days last year were 20, 40, 50, 60, and 80.

Determine the mean deviation for the number of cappuccinos sold.

LO7

EXAMPLE – Mean Deviation

The number of cappuccinos sold at the Starbucks location in the Orange Country Airport between 4 and 7 p.m. for a sample of 5 days last year were 20, 40, 50, 60, and 80.

Determine the mean deviation for the number of cappuccinos sold.

Step 1: Compute the mean

$$\bar{x} = \frac{\sum x}{n} = \frac{20+40+50+60+80}{5} = 50$$

EXAMPLE – Mean Deviation

Step 2: Subtract the mean (50) from each of the observations, convert to positive if difference is negative

Step 3: Sum the absolute differences found in step 2 then divide by the number of observations

Number of Cappuccinos Sold Daily	$(X - \bar{X})$		Absolute Deviation
20	$(20 - 50) = -30$		30
40	$(40 - 50) = -10$		10
50	$(50 - 50) = 0$		0
60	$(60 - 50) = 10$		10
80	$(80 - 50) = 30$		30
		Total	80

$$MD = \frac{\Sigma|X - \bar{X}|}{n} = \frac{80}{5} = 16$$

Variance and Standard Deviation

VARIANCE The arithmetic mean of the squared deviations from the mean.

STANDARD DEVIATION The square root of the variance.

- The variance and standard deviations are nonnegative and are zero only if all observations are the same.
- For populations whose values are *near the mean*, the variance and standard deviation will be small.
- For populations whose values are *dispersed from the mean*, the population variance and standard deviation will be large.
- The variance overcomes the weakness of the range by using all the values in the population

LO8

Variance – Formula and Computation

POPULATION VARIANCE $$\sigma^2 = \frac{\Sigma(X-\mu)^2}{N}$$ **[3–8]**

σ^2 is the population variance (σ is the lowercase Greek letter sigma). It is read as "sigma squared."
X is the value of an observation in the population.
μ is the arithmetic mean of the population.
N is the number of observations in the population.

Steps in Computing the Variance.

Step 1: Find the mean.
Step 2: Find the difference between each observation and the mean, and square that difference.
Step 3: Sum all the squared differences found in step 2
Step 4: Divide the sum of the squared differences by the number of items in the population.

LO8

EXAMPLE – Variance and Standard Deviation

The number of traffic citations issued during the last year by month in Beaufort County, South Carolina, is reported below:

Month	January	February	March	April	May	June	July	August	September	October	November	December
Citations	19	17	22	18	28	34	45	39	38	44	34	10

What is the population variance?

EXAMPLE – Variance and Standard Deviation

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What is the population variance?

Step 1: Find the mean.

$$\mu = \frac{\sum x}{N} = \frac{19+17+...+34+10}{12} = \frac{348}{12} = 29$$

Step 2: Find the difference between each observation and the mean, and square that difference.

Step 3: Sum all the squared differences found in step 3

Step 4: Divide the sum of the squared differences by the number of items in the population.

LO8

EXAMPLE – Variance and Standard Deviation

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What is the population variance?

Step 2: Find the difference between each observation and the mean, and square that difference.

Step 3: Sum all the squared differences found in step

Step 4: Divide the sum of the squared differences by the number of items in the population.

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{1{,}488}{12} = 124$$

Month	Citations (X)	$X - \mu$	$(X - \mu)^2$
January	19	−10	100
February	17	−12	144
March	22	−7	49
April	18	−11	121
May	28	−1	1
June	34	5	25
July	45	16	256
August	39	10	100
September	38	9	81
October	44	15	225
November	34	5	25
December	10	−19	361
Total	348	0	1,488

LO8

Sample Variance

SAMPLE VARIANCE $$s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1}$$ **[3–10]**

Where :

s^2 is the sample variance

X is the value of each observatio n in the sample

$\overline{X}$ is the mean of the sample

n is the number of observatio ns in the sample

EXAMPLE – Sample Variance

The hourly wages for a sample of part-time employees at Home Depot are: $12, $20, $16, $18, and $19.

What is the sample variance?

LO8

EXAMPLE – Sample Variance

The hourly wages for a sample of part-time employees at Home Depot are: $12, $20, $16, $18, and $19.

What is the sample variance?

SAMPLE VARIANCE $$s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1}$$ [3–10]

Hourly Wage (X)	$X - \overline{X}$	$(X - \overline{X})^2$
$12	−$5	25
20	3	9
16	−1	1
18	1	1
19	2	4
$85	0	40

$$s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1} = \frac{40}{5 - 1}$$

$$= 10 \text{ in dollars squared}$$

LO8

Sample Standard Deviation

SAMPLE STANDARD DEVIATION

$$s = \sqrt{\frac{\Sigma(X - \overline{X})^2}{n - 1}} \qquad [3\text{–}11]$$

Where :

s^2 is the sample variance

X is the value of each observation in the sample

$\overline{X}$ is the mean of the sample

n is the number of observations in the sample

The Empirical Rule

EMPIRICAL RULE For a symmetrical, bell-shaped frequency distribution, approximately 68 percent of the observations will lie within plus and minus one standard deviation of the mean; about 95 percent of the observations will lie within plus and minus two standard deviations of the mean; and practically all (99.7 percent) will lie within plus and minus three standard deviations of the mean.

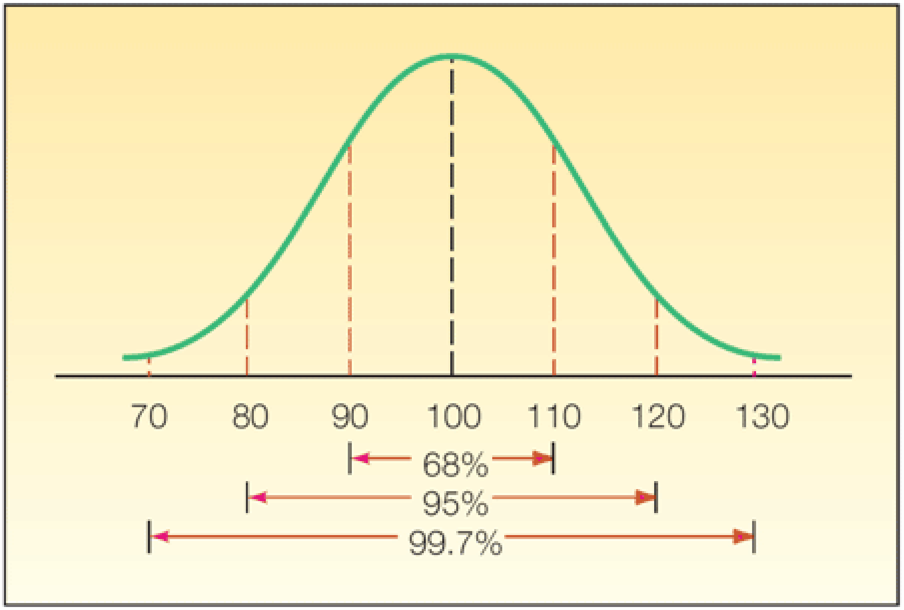

CHART 3–7 A Symmetrical, Bell-Shaped Curve Showing the Relationships between the Standard Deviation and the Observations

LO10 Compute the mean and standard deviation of grouped data.

The Arithmetic Mean of Grouped Data

ARITHMETIC MEAN OF GROUPED DATA $$\bar{X} = \frac{\Sigma fM}{n}$$ [3–12]

where:

- $\bar{X}$ is the designation for the sample mean.
- M is the midpoint of each class.
- f is the frequency in each class.
- fM is the frequency in each class times the midpoint of the class.
- ΣfM is the sum of these products.
- n is the total number of frequencies.

LO10

The Arithmetic Mean of Grouped Data - Example

Recall in Chapter 2, we constructed a frequency distribution for Applewood Auto Group profit data for 180 vehicles sold. The information is repeated on the table. Determine the arithmetic mean profit per vehicle.

Profit	Frequency
$ 200 up to $ 600	8
600 up to 1,000	11
1,000 up to 1,400	23
1,400 up to 1,800	38
1,800 up to 2,200	45
2,200 up to 2,600	32
2,600 up to 3,000	19
3,000 up to 3,400	4
Total	180

The Arithmetic Mean of Grouped Data - Example

Profit	Frequency (f)	Midpoint (M)	fM
$ 200 up to $ 600	8	$ 400	$ 3,200
600 up to 1,000	11	800	8,800
1,000 up to 1,400	23	1,200	27,600
1,400 up to 1,800	38	1,600	60,800
1,800 up to 2,200	45	2,000	90,000
2,200 up to 2,600	32	2,400	76,800
2,600 up to 3,000	19	2,800	53,200
3,000 up to 3,400	4	3,200	12,800
Total	180		$333,200

Solving for the arithmetic mean using formula 3-12, we get:

$$\overline{X} = \frac{\Sigma fM}{n} = \frac{\$333,200}{180} = \$1,851$$

LO10

Standard Deviation of Grouped Data - Example

Refer to the frequency distribution for the Applewood Auto Group data used earlier. Compute the standard deviation of the vehicle profits.

Profit	Frequency (f)	Midpoint (M)	fM	$(M - \overline{X})$	$(M - \overline{X})^2$	$f(M - \overline{X})^2$
\$ 200 up to \$ 600	8	400	3,200	−1,451	2,105,401	16,843,208
600 up to 1,000	11	800	8,800	−1,051	1,104,601	12,150,611
1,000 up to 1,400	23	1,200	27,600	−651	423,801	9,747,423
1,400 up to 1,800	38	1,600	60,800	−251	63,001	2,394,038
1,800 up to 2,200	45	2,000	90,000	149	22,201	999,045
2,200 up to 2,600	32	2,400	76,800	549	301,401	9,644,832
2,600 up to 3,000	19	2,800	53,200	949	900,601	17,111,419
3,000 up to 3,400	4	3,200	12,800	1,349	1,819,801	7,279,204
Total	180		333,200			76,169,780

$$s = \sqrt{\frac{\Sigma f(M - \overline{X})^2}{n - 1}} = \sqrt{\frac{76,169,780}{180 - 1}} = 652.33$$